Phase synchrony in neurofeedback protocols

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Online estimation of phase synchrony measures in EEG is subject to a number of pitfalls, which makes such neurofeedback protocols rare or inadequate, despite their potential in health care applications (epilepsy syndromes, attention deficit hyperactivity disorders...).
We discuss three crucial points for using phase synchrony measures in EEG neurofeedback protocols : phase estimation mixes close frequency components, timely close phase estimations are statistically correlated, synchrony measures are redundant and / or complementary.

A] Phase estimation mixes close frequency components
For phase estimation, a narrow band filtering followed by a Hilbert transform, a windowed Fourier Transform or a Morlet Wavelet Transform hold fundamentally equivalent results under the right set of parameters ([1] and [2]).

We derive the Morlet Wavelet Transform of a signal ($f(t)$)

$$WT(f(t))(k) = \frac{1}{\sqrt{\frac{\sqrt{\pi}\,\Omega}{\omega_w}}} \int_{-\infty}^{\infty} f(k-t) \mathrm{e}^{\mathrm{i}\omega_w t} \mathrm{e}^{-\frac{t^2\omega_w^2}{2\Omega^2}}\, dt$$

where Ω : number of oscillations, $\omega_w = 2\pi\nu_w$: frequency.

For a simple sinusoidal function $\cos(\omega_c t + \phi_c)$, we obtain :

$$\phi(k) = arctan(\frac{\Im(WT(f(k)))}{\Re(WT(f(k)))}) = arctan(tan(\omega_c k + \phi_c) tanh(\frac{\Omega^2\omega_c}{\omega_w}))$$

We observe that the width of the wavelet has to be bigger than two oscillations ($tanh(\Omega^2) \approx 1$ for $\Omega > 2$ for $\omega_w = \omega_c$) to avoid phase wrapping, regardless of the frequency.
The only consequence of modifying the $tanh$ factor will be to warp the phase, not shift it.

We then generalized it to a signal approximated by a sum of sinusoidal functions ($\alpha_i \cos(\omega_i t + \phi_i)$) locally stationary (α_i constant).

We obtain an expression of the "phase" for multi-frequency signals:

$$\phi(k) = arctan \left(\frac{\sum \alpha_i \left(\mathrm{e}^{-\frac{\Omega^2(\omega_i-\omega_w)^2}{2\omega_w^2}} - \mathrm{e}^{-\frac{\Omega^2(\omega_i+\omega_w)^2}{2\omega_w^2}} \right) sin(\omega_i k + \phi_i)}{\sum \alpha_i \left(\mathrm{e}^{-\frac{\Omega^2(\omega_i-\omega_w)^2}{2\omega_w^2}} + \mathrm{e}^{-\frac{\Omega^2(\omega_i+\omega_w)^2}{2\omega_w^2}} \right) cos(\omega_i k + \phi_i)} \right)$$

We study the influence of the coefficients $\alpha_i \left(\mathrm{e}^{-\frac{\Omega^2(\omega_i-\omega_w)^2}{2\omega_w^2}} - \mathrm{e}^{-\frac{\Omega^2(\omega_i+\omega_w)^2}{2\omega_w^2}} \right)^*$ on the weighting of each sinusoidal component :
1) The coefficients are symmetric in the frequency of the signal with respect to the frequency of the wavelet.
2) Increasing the width of the wavelet is an option to increase the frequency accuracy of the phase estimation : it minimizes close frequency components mixing (Figure, a).
3) The benefit of increasing the width of the wavelet (Ω) to values higher than 30 is minimal, and at the expanse of reducing the temporal accuracy.(Figure, a)
4) We show that the higher the frequency of interest is, the more the mixing between frequency components around that frequency (respecting the

uncertainty principle) (Figure, b).
5) The amplitude ratio between components matters in the phase estimation, but lightly in comparison to the distance ratio between the frequency of the wavelet and the frequency of the components.

B] Timely close phase estimations are statistically correlated
We show, assuming an EEG signal described by a sequence of random variables with identical mean and identical variance, that two close wavelet coefficients are correlated, and so are the phase estimations.
We prove that the correlation coefficient between two ∂t distant Morlet Wavelet coefficients in this setup is equal to the ∂t lagged correlation of the Morlet Wavelet.
The statistical assumption of independence between coefficients is only true for non overlapping wavelet windows.
The estimators (as in [3]) hypothesizing independence must be corrected to be used with overlapping Morlet Wavelet windows.
This correction will allow for an accurate estimation of the potential error made on the synchrony given a sample size and overlapping size, a work not presented here.

C] Synchrony measures are redundant and / or complementary
The phase synchrony measures (Phase Lag Index, Phase Locking Value, Coherence, Imaginary Coherence...) and their properties (stability, reliability, sensitivity to volume conduction), despite having the same goal (assessing synchrony), behave differently in practice.
We study and compare the linear relationships between these commonly used synchrony measures on simple task recordings, to see that some are redundant while others are complementary.

*coefficients of the denominator behave similarly for $\Omega > 3$, and so the mixing between components, hence we only address the numerator.

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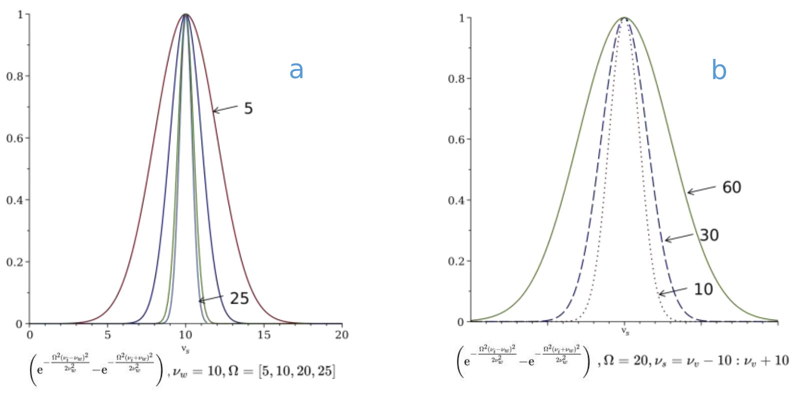

Figure